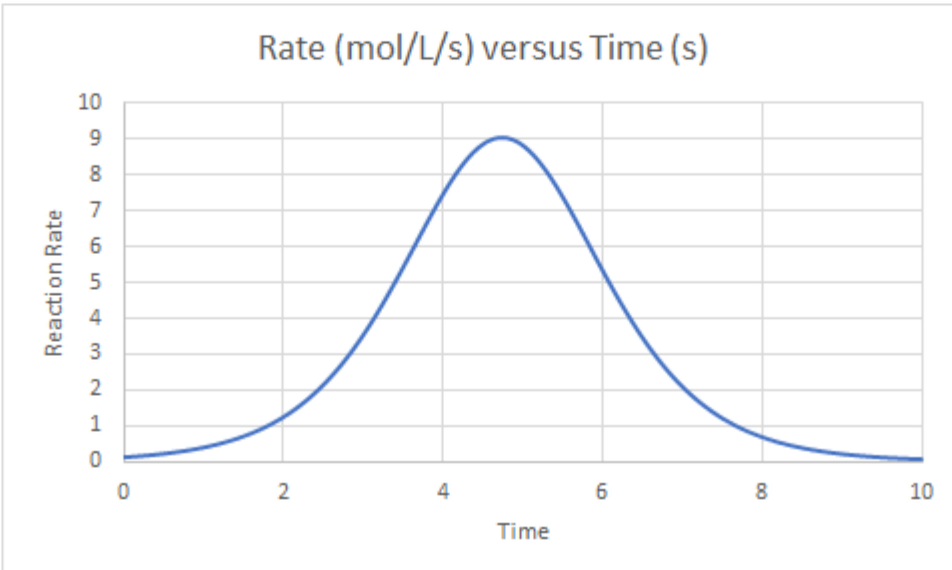
WUCT: Team Exam Sample Questions

1. Autocatalysis is a process by which a small amount of reactant is able to catalyze another reactant to form twice the amount of catalyst. The general overall reaction is represented below:

$$A + B \rightarrow 2B$$

B catalyzes A into B. One real life example of this is the conversion of β-Tin to α-Tin via an autocatalytic, allotropic transformation. α-Tin, known as tin pest, is able to convert β-Tin into more α-Tin. This process usually begins with a small conversion to α-Tin at a corner of a sheet of β-Tin and subsequently leads to expansion of α-Tin via autolytic conversion.



a. The graph above displays the reaction rate of the general equation presented above. Describe the general curvature the graph assumes in context to the reaction. Specifically, focus on the catalysis of A by B.
The initial portion of the graph, from 0 to 2.5 seconds describes the slow increase of B as it consumes A. The initial rate of B catalyzing A is slow due to the low quantity of B. As B begins to grow, the graph assumes a steeper curvature, between 2.5 and 5.5. This process describes the large amount of B catalyzing a similarly large quantity of A. The peak in the graph describes the max rate of B catalyzing A. The ratio of B to A depends on the kinetics and thermodynamics of the reaction; an optimal ratio produces the peak. As B grows exponentially, A rapidly decreases, between 5.5 and 6.5. The final stage of the reaction consists of a large amount of B and a small amount of A, therefore the change in rate is slow because the B is not converting a large amount of A as before.

b. Write two differential rate laws with respect to the loss of A and the production of B. Assume the rate constant to be **k** and the reaction to follow an elementary mechanism.

$$Loss\ of\ A: \frac{dA}{dt} = -k[A][B]$$
$$Loss\ of\ B: \frac{dB}{dt} = 2k[A][B]$$

c. There are three primary phases of an autocatalytic reaction: lag, log and stationary. Will the autocatalytic reaction have multiple rate constants or a single rate constant? Explain.
When looking at the a rate equation it is composed of temperature dependent terms (Rate constant) and concentration dependent terms (in this case $[A][B]$). Since the overall reaction scheme remains constant, the rate constant will remain constant. Additionally, the total concentrations of both reactant A and the autocatalyst B remain conserved. If a third autocatalyst C were to be introduced, then the reaction would proceed in a stepwise fashion.

2. Electrons have both particle and wave-like behavior. The particulate behavior of electrons can be described by De Broglie's equation relating momentum and wavelength.

$$p = \frac{h}{\lambda}$$

Where $p =$ mv (mass times velocity), h is Planck's constant ($6.626 \cdot 10^{-34}\ Js$), and λ represents the wavelength.

a. Describe what happens to the momentum as the wavelength decreases. How does this relate to the frequency of the wave and the kinetic energy?

As the wavelength decreases, the momentum of the particle increases. The wavelength describes the distance between wave peaks and the frequency describes the number of oscillations within a certain change in time. As the wavelength decreases, the frequency will also increase. The frequency is directly related to energy, as frequency increases, as does energy. More theoretically, momentum assumes the form mv and kinetic energy assumes the form 0.5 mv^2. Aside from the exponential growth of velocity and constant 0.5, the momentum and kinetic energy have a direct relationship. As the wavelength decreases, the momentum increases, and therefore, as does the kinetic energy.

b. Given the mass of an electron is $9.11 \cdot 10^{-31}\ kg$ and is moving at 70% the speed of light ($c = 3.0 \cdot 10^{8}\ {}^{m}/_{s}$), what is the momentum, wavelength and frequency (Hz) of the electron?

First, we must calculate the velocity of the traveling wave, which is:

$$\boldsymbol{\vec{v} = (0.01)\left(3.0 \cdot 10^{8}\right) = 3.0 \cdot 10^{6}\ {}^{m}/_{s}}$$

The momentum can be calculated using the formula: $\boldsymbol{\vec{p} = m\vec{v}}$

$$\boldsymbol{\vec{p} = \left(9.11 \cdot 10^{-31}\right)\left(3.0 \cdot 10^{6}\right) = 2.733 \cdot 10^{-24}\ {}^{kg \cdot m}/_{s}}$$

The wavelength can be found using $\boldsymbol{\lambda = h/\vec{p}}$

$$\boldsymbol{\lambda = \frac{(6.626 \cdot 10^{-34})}{(2.733 \cdot 10^{-24})} = 2.424 \cdot 10^{-10} m}$$

The frequency can be found using a simple relationship $\boldsymbol{\lambda \nu = c}$

(Note that the velocity was represented as a vector and frequency as scalar despite using the same symbol)

$$\boldsymbol{\nu = \frac{(3.0 \cdot 10^{8})}{(2.424 \cdot 10^{-10})} = 1.237 \cdot 10^{18} Hz}$$

The wavelength is very close to the size of an atom. The electron moving around the nucleus describes an electric charge essentially oscillating to produce electromagnetic waves. This is why the wavelength is so similar to the atomic size. Additionally, the wavelength is observable to a degree and sets the foundation for many quantum mechanical phenomena.

c. A baseball weighs 0.145 kg and is moving at a velocity of 40 m/s. Calculate the wavelength of the baseball. The correspondence principle states that we can reproduce quantum mechanical phenomena on a classical scale upon the increase of the principal quantum number. In context to the correspondence principle, compare the wavelength of a baseball and an electron and why we don't observe baseballs "teleporting" from one location to another.

In order to calculate the wavelength, we must calculate the momentum. Using the equation above, the momentum is $\mathbf{5.8}\,{}^{\boldsymbol{kg \cdot m}}\!/_{\boldsymbol{s}}$. The wavelength will therefore be $\mathbf{1.14 \cdot 10^{-34}}\boldsymbol{m}$.

This wavelength is incredibly small and this is the primary reason we don't observe the wave-like properties in classical physics. In this case, we drastically increased the mass of the object and found that the system itself can be observed to obey the laws of classical physics yet theoretically still preserves the laws of quantum mechanics. This increase in size and discrepancy in observation is due to the correspondence principle. Since the baseballs wavelength is incredibly small compared to that of the electrons, we observe two different fundamental properties.

d. Heisenberg's uncertainty principle states that we cannot measure both position and momentum with absolute precision in a quantum mechanical system. If we know that the momentum of a particle in free space is known with absolute precision, what is the probability of finding the particle at a certain location in space? What is the probability of finding the particle in all space?

The probability of finding the particle in an area of space will be some arbitrary quantity, however, it will be constant throughout the entire space. This entails that finding the particle in a certain region is indeterminate. This DOES NOT, however, mean that the particle must have 0 probability of being found, otherwise, the particle itself will not exist. The probability of finding the particle in all space is 1. The particle must be somewhere within the space, and therefore, has a 100 percent probability of being in the space.

e. The wavelike characteristics of quantum mechanical particles is often encapsulated within wavefunctions. Wavefunctions describe the probability of finding a particle in a certain location. The wavefunction for a particle on a ring (a fundamental model for simplifying the hydrogen atom) is presented below. The probability of a particle being somewhere in space can be described by multiplying the wavefunction by its complex conjugate. A complex conjugate simply inverses any complex numbers (i becomes -i and vice versa). Given the wavefunction, describe the probability of the particle in space. What does this tell us about the angular momentum of the particle?

$$\psi(\phi) = \left(\sqrt{\frac{1}{2\pi}}\right) e^{im\phi}$$

We can start by multiplying the wavefunction by its complex conjugate. The bar is not necessary; it simply denotes the complex conjugate of the wave function.

$$P = \psi(\phi) \cdot \overline{\psi(\phi)} = \left(\sqrt{\frac{1}{2\pi}}\right) e^{im\phi} \cdot \left(\sqrt{\frac{1}{2\pi}}\right) e^{-im\phi} = \frac{1}{2\pi}$$

The complex exponentials cancel and we are left with the multiplication of two constants, giving the solution above. The probability of finding the particle somewhere in space is constant, indicating that the probability of finding the particle is completely uncertain throughout space. Therefore, we must know the momentum with absolute precision. In this scenario, we know the angular momentum with absolute precision and the position around the ring with complete uncertainty.